विद्या सर्वार्थि साधिका

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SUMMATIVE ASSESSMENT - 1

Class XII

Subject: MATHEMATICS M.M: 100

Date : 28/ 09/ 2016 Time: 3 Hours

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Questions **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Questions **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Questions **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Questions **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

Section-A

1. Evaluate: $\int \frac{sec^2\sqrt{x}}{\sqrt{x}}dx$

2. If y = $\sin(x^x)$, find $\frac{dy}{dx}$.

3 Given $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, and $A.adj\ A = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find k.

4. Find whether the relation R in the set A = {1, 2, 3} given by
R = {(1, 1), (1, 2), (2, 2), (2, 3), (3, 3)} is transitive.

Section-B

5. Find the value of $tan^{-1}\left[2cos\left(2sin^{-1}\frac{1}{2}\right)\right]$.

6. Show that the function $f: R \to R \quad defined\ by\ f(x) = |x|$ is neither one –one nor on to.

7. Without expanding, prove that: $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix} = 0$

8. A and B are symmetric matrices, show that $AB + BA$ is symmetric and $AB - BA$ is skew-symmetric.

9. Show that the function $f(x) = |x - 3|,\ x \in R$ is continuous but not differentiable at x = 3.

10. The radius r cm of a blot of ink is increasing at the rate of 1.5 mm/sec. Find the rate at which the area A is increasing after 4 sec.

11. Evaluate: $\int tan^3x\ dx$.

12. Evaluate: $\int e^x\ \frac{x^2+1}{(x+1)^2}dx$.

Section-C

13. Prove that : $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

14. Let R be a relation defined on Z such that $R = \{(a,b) : a,b \in Z, and\ |a-b| \leq 5\}$. Check whether R is i) Reflexive ii) Symmetric iii) Transitive.

15 Using properties of determinants, show that : $\begin{vmatrix} (x+1)(x+2) & x+2 & 1 \\ (x+2)(x+3) & x+3 & 1 \\ (x+3)(x+4) & x+4 & 1 \end{vmatrix} = -2$

16. If the function f defined by $f(x) = \begin{cases} \frac{\sin(a+1)x + sinx}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - x}{bx^{3/2}}, & x > 0 \end{cases}$ is continuous at $x = 0$. Then find the values of a, b and c.

OR

If $\log(x^2 + y^2) = 2\tan^{-1}\frac{y}{x}$ then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

17. Find the equation of the tangent to the curve $y = cot^2 x - 2\cot x + 2$ at $x = \frac{\pi}{4}$.

OR

Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x – axis.

18. Using differentials find approximate value of $255^{1/4}$ up to three decimals.

19. Find the intervals in which the function $f(x) = \frac{x}{2} + \frac{2}{x}, x \neq 0$ is strictly increasing or decreasing.

20. If $A = \begin{bmatrix} 2 & 3 \\ 5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 6 \\ 11 & 8 \end{bmatrix}$, find a matrix C, such that $2A + 3B + 4C$ is identity matrix.

OR

A typist charges Rs. 145 for typing10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs. 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only Rs.2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this boy? **Which values are reflected in this problem?**

21. If $x = asec^3\theta\ and\ y = atan^3\theta, find\ \frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

22. Evaluate : $\int_0^{\frac{\pi}{4}} \frac{sinx\ cosx}{sin^4x + cos^4x}\, dx$.

23. Evaluate: $\int \frac{x^3+x+1}{x^2-1}\, dx$.

Section-D

24 Prove that the product of matrices

$$\begin{bmatrix} cos^2\theta & \frac{\sin 2\theta}{2} \\ \frac{\sin 2\theta}{2} & sin^2\theta \end{bmatrix} \quad and \quad \begin{bmatrix} cos^2\emptyset & \frac{\sin 2\emptyset}{2} \\ \frac{\sin 2\emptyset}{2} & sin^2\emptyset \end{bmatrix}$$

is null matrix when $\theta - \emptyset$ is an odd multiple of $\frac{\pi}{2}$.

25. Differentiate $tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ $w.r.t\ cos^{-1}x^2$.

OR

Prove that the derivative of $tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $tan^{-1}x$ is independent of x.

26. An open box with a square base is to be made out of a given quantity of metal sheet of area c^2.

Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also, find the maximum volume

27. Using properties of definite integrals, show that: $\int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x}\, dx = \frac{\pi^2}{2ab}$.

OR

Evaluate: $\int_1^4 \{|x-1| + |x-2| + |x-3|\}dx$.

28 Let A = N × N and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (ad + bc, bd)$ for all $(a, b), (c, d) \in N \times N$.
Show that $*$ is (i) commutative (ii) associative iii) A has no identity element in A.

29. Evaluate: $\int_0^3 (3x^2 - 4)\, dx$ as limit of sums.